BRAILLE MATHEMATICAL NOTATION

By H. M. TAYLOR

SECOND AMERICAN REVISION 1942

Compiled by

JOHN B. CURTIS, Founder and Former Supervisor Classes for the Blind, Chicago Public Schools

> MARJORIE S. HOOPER, Braille Editor American Printing House for the Blind

EDWARD J. WATERHOUSE, Instructor in Mathematics
Perkins Institution and Massachusetts School
for the Blind

Published in both Braille and Ink Print

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BRAILLE MATHEMATICAL CODE may be

obtained from the

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PREFACE

The 2nd American Revision of the BRAILLE MATHEMATICAL NOTA-TION was undertaken because of the inability of the previous revision to meet the demands of modern mathematics texts, and because it was necessary to bring the Mathematics Code in line with Standard English Braille. This revision is based on the code devised by H. M. Taylor, and revised in 1920 by the Sub-Committee on Mathematics of the Commission on Uniform Type for the Blind (John B. Curtis, Chairman; with Claudia Potter and Nellie G. McIntyre); accepted by the Commission on Uniform Type, June 23, 1920; and referred for consideration and approval to the International Sub-Committee on Uniform Type.

In making this revision, two purposes have been kept in mind: One, the need for a manual to aid in the teaching of the Braille Mathematics Code; and Two, the need for a comprehensive guide for stereotypists, transcribers and proof-readers of Braille mathematical works.

The order of the introduction of the mathematical symbols is designed to follow the order of their introduction in arithmetic, algebra, geometry, and higher mathematics. This order was re-

tained from the previous revision with very little change.

The arrangement of the material has been changed from paragraph to outline form, and the entire format expanded to increase ease of identification of the symbols. The Braille edition is embossed in one-side print, and the pagings of the Braille edition have been indicated in the ink-print edition, for the convenience of sighted instructors.

A number of additional illustrations of usage of the symbols have been included, and the list of abbreviations has been increased. Many of the usages noted are not recommended for common use, but represent forms which the student may find in Braille mathematical textbooks embossed prior to this revision or in other countries. An attempt has been made to indicate the preferred usage.

The inclusion of a Table of Contents and the change to outline arrangement should greatly increase the usability of the NOTATION by embossers and proofreaders.

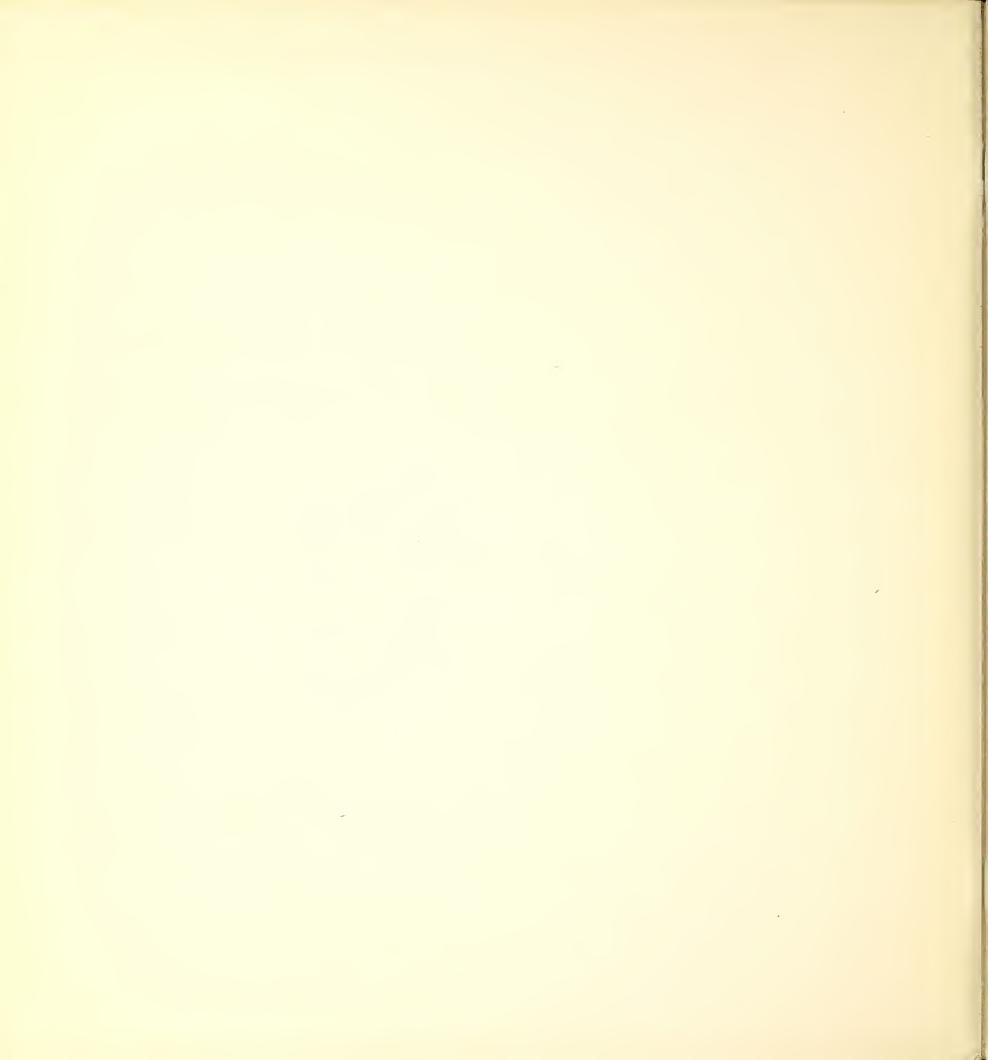
October, 1942

JOHN B. CURTIS
MARJORIE S. HOOPER
EDWARD J. WATERHOUSE

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MATHEMATICAL NOTATION

(Note: Bold-face numbers inclosed in brackets indicate the pagings of the Braille edition of Mathematical Notation.)

ARBITRARY SYMBOLS AND THEIR USES

NUMERALS

Numeral Sign : (dots 3-4-5-6). Numerals are expressed by placing the Numeral Sign immediately before the first ten signs of the alphabet:

Cardinal Numbers are written thus:

In numbers of more than three figures, except when writing the date of the year, the Comma : (dot 2) is used to divide off the thousands. This rule is frequently disregarded in numbers less than 10,000, and the Comma should also be omitted from computation examples if its inclusion throws figures into the [2] wrong column. Examples:

Numbers joined by the **Hyphen** do not require a second **Numeral Sign** unless the number is divided at the end of a line after the **Hyphen** when the **Numeral Sign** should be repeated at the beginning of the following line. Examples:

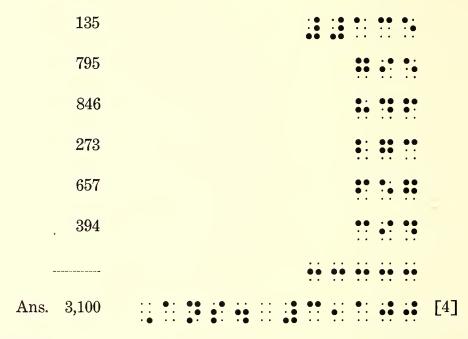
When an Integral Number is divided at the end of a line, the Hyphen is placed either at the end of the first line or at the beginning of the second, or preferably both, and the Numeral Sign should not be repeated. Example:

```
-810
```

The Numeral Sign is frequently omitted from mathematical tables without [3] explanation when its presence may easily be assumed. It should not, however, be omitted after a sign of operation in cases where the various parts of a problem are written on separate lines. For illustrations, see examples under the General Sign of Operation, page 12.

Double Numeral Sign : (dots 3-4-5-6, 3-4-5-6) indicates that the Numeral Sign is omitted from all the numbers of the immediate group. For example:

Find the sum of the following: 135, 795, 846, 273, 657, and 394. Solution:



Considerable discretion should be employed in the use of the **Double Number Sign.** It should not be used unless more than two numbers are involved. It should also not be used in books below the level of the fourth grade

Note: The use of the continuous line of dots 2-5, 2-5, etc., etc., to indicate the addition of the figures in the columns above is authorized by the General Sign of Operation, page 12.

Triple Numeral Sign : (dots 3-4-5-6, 3-4-5-6, 3-4-5-6) indicates that all of the Braille signs represented by the first ten letters of the alphabet which follow on the page, or until the Close Triple Numeral Sign (dots 1-4-5-6, 1-4-5-6, 1-4-5-6) is met, represent Arabic numerals. (Note: These signs are seldom employed.) [5]

Ordinal Numbers are written by placing the proper ending immediately following the Cardinal Number without a space. Braille contractions should be employed for the suffix according to the grade of Braille being used. Examples:

It is recommended that the forms "nd" and "rd" should be used instead of "d" to prevent confusion in reading the suffix as "the Cardinal Number 4."

Roman Numerals of one letter are preceded by the Capital Sign :: (dot 6); those of more than one letter, by the Double Capital Sign :: [6] (dots 6, 6). For example:

LETTERS

Letter Sign. (dots 5-6). Single letters, when representing algebraic quantities, are preceded by the Letter Sign. Contrary to literary Braille usage, it is suggested that this rule also apply to the writing of the letter "a" in mathematical Braille. Examples:

```
a :: :: x :: :: y :: :: :: :: y
```

When the letter is preceded or followed immediately by another character, the Letter Sign is omitted if it is perfectly clear that the letter cannot be interpreted as a number. Examples:

```
3x : xy : xy : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3 : 3
```

In algebra, whenever a combination of letters may be taken for a word, it is recommended that the Letter Sign precede such a combination. [7] Examples:

```
ab (which might be taken for about)ancd (for could)
```

When an algebraic quantity represented by one of the first ten letters of the alphabet has a numerical coefficient, the Multiplication Sign (see page 11) follows the number, that is, it follows the coefficient. For example:

See also rules for Mixed Numbers, page 18; Literal Index Sign, page 20; Literal Suffix Sign, page 22; and Factorial Sign, page 28.

Greek Letters are formed by placing dot 3 :: immediately before corresponding English letters.

For example:

Delta		Upsilon :: •:
Epsilon		Phi :: •••
Eta		Chi
Theta	[8]	Psi ::
Omicron		Omega :: :•
Pi	· · • • · · · • · · · · · · · · · · · ·	(For a complete list of Greek Letters, see page 37.)

Geometric Points are usually expressed by Small Letters in Braille, contrary to the ink-print practice of using Capital Letters, except where the same letter is used in both capital and small forms in the same exercise.

DECIMALS

Decimal Point . : (dots 4-6). When a decimal fraction is joined to a whole number, only one Numeral Sign (that before the whole number) is required. Examples:

SYMBOLS OF VALUE, MEASUREMENT, ETC.

When in ink print a symbol or a literal abbreviation of value or measurement follows a numeral, the corresponding literal abbreviation or its equivalent, without the Abbreviation Point, is placed in Braille before the Numeral Sign.

American Money:

Dollar Sign \$: (dots 2-5-6, 3-4-5-6). In decimal coinage the Decimal Point is used in place of the second Numeral Sign. [10] Examples:

Note: In books embossed prior to the adoption of Standard English Braille in 1932, the sign (dots 1-4-5, 3-4-5-6) is used to express the Dollar Sign.

Cent Sign ¢ : (dots 1-4, 3-4-5-6) should only be used to express amounts less than a dime, including the decimal parts of a cent, or when indicated in the ink-print text. The sign for Mills : (dots 1-3-4, 3-4-5-6) is seldom used. Examples:

 9ϕ

 $.015\phi$ rather than \$.00015 [11]

Foreign Money:

Pound Sterling £ \vdots (dots 1-2-3, 3-4-5-6).

Shilling / or s : (dots 2-3-4, 3-4-5-6).

Pence d : (dots 1-4-5, 3-4-5-6).

Guinea gs ... (dots 1-2-4-5, 2-3-4, 3-4-5-6).

Franc fr : (dots 1-2-4, 1-2-3-5, 3-4-5-6)

Mark (Reich) rm (dots 1-2-3-5, 1-3-4, 3-4-5-6).

Peso P •• : (dots 1-2-3-4, 3-4-5-6). [12]

Rupee rs (dots 1-2-3-5, 2-3-4, 3-4-5-6).

Yen Y : (dots 1-3-4-5-6, 3-4-5-6).

Miscellaneous Symbols:

Grams gm : (dots 1-2-4-5, 1-3-4).

```
Acres a :: (dot 1).
Barrels bbl :: :: (dots 1-2, 1-2, 1-2-3).
Bushels bu :: (dots 1-2, 1-3-6).
Centimeters cm ... (dots 1-4, 1-3-4).
Cubic Inches cu. in. ... (dots 1-4, 1-3-6, 3-5-6, 2-4, 1-3-4-5). This same form can be used in writing "cubic feet," "cubic yards," etc.
Days da ... (dots 1-4-5, 1).
Decimeters dm : (dots 1-4-5, 1-3-4).
Degrees ° (dots 1-4-5, 1-2-4-5). The Degree Sign may [13] be used to indicate measurements of
   arcs and angles as well as of temperatures. Examples:
       10° .....
        Dozen dz : (dots 1-4-5, 1-3-5-6).
Feet ft ' :: (dots 1-2-4, 2-3-4-5).
Gallons gal :: (dots 1-2-4-5, 1, 1-2-3).
Gills gi . (dots 1-2-4-5, 2-4).
```

```
Gross gro : (dots 1-2-4-5, 1-2-3-5, 1-3-5).
Hours hr (dots 1-2-5, 1-2-3-5).
Hundred Weight cwt ... (dots 1-4, 2-4-5-6, 2-3-4-5).
Inches in " . . . (dots 2-4, 1-3-4-5). The contracted form should never be used in mathematical
   Braille. [14].
Kilograms kg :: •• (dots 1-3, 1-2-4-5).
Kilometers km :: •• (dots 1-3, 1-3-4).
Meters m ... (dots 1-3-4).
Miles mi . . . (dots 1-3-4, 2-4).
Millimeters mm . . . (dots 1-3-4, 1-3-4).
Minutes min ' : : : (dots 1-3-4, 2-4, 1-3-4-5) or : (dots 1-3-4, 1-3-4-5). The former is the pre-
   ferred usage, and should never be written in contracted form in mathematical Braille. This sign
   may be used to indicate minutes of angles and arcs as well as of time.
Months mo : : (dots 1-3-4, 1-3-5).
Ounces oz : (dots 1-3-5, 1-3-5-6).
Pecks pk • • • (dots 1-2-3-4, 1-3).
Per Cent Sign % ... (dots 2-5, 1-2-3-4). Examples: [15]
       5% :: :: ::
```

```
Pints pt (dots 1-2-3-4, 2-3-4-5).
Pound Weight # or lb : .: (dots 1-2-3, 1-2).
Quarters qr (dots 1-2-3-4-5, 1-2-3-5).
Quarts qt (dots 1-2-3-4-5, 2-3-4-5).
Rods rd (dots 1-2-3-5, 1-4-5).
Seconds " . . (dots 2-3-4, 1-4). The sign for Seconds may be used to indicate seconds of angles and
    arcs as well as of time.
Square Feet sq. ft. (dots 2-3-4, 1-2-3-4-5, 2-5-6, 1-2-4, 2-3-4-5). This same form can be
    used in writing "square inches," "square yards," etc.
Tablespoons tbsp : : : : (dots 2-3-4-5, 1-2, 2-3-4, 1-2-3-4). [16]
Teaspoons tsp : (dots 2-3-4-5, 2-3-4, 1-2-3-4).
Tons t (dots 2-3-4-5).
Weeks wk : (dots 2-4-5-6, 1-3).
Yards yd : (dots 1-3-4-5-6, 1-4-5).
Years yr : (dots 1-3-4-5-6, 1-2-3-5).
```

Additional Symbols of Measurements may be developed by placing the accepted abbreviation authorized by any standard dictionary before the numeral sign without a space, as in the examples given above. In this connection, it should be noted that the form . (dots 1-4) cannot be used for "cups" or "cans," since in Braille this character represents "cents." It is also suggested that no abbreviation be used for "section," since this is not often needed and might be confused with the abbreviation for [17] "seconds."

Usages:

In a sequence of related terms, only the Abbreviation for the larger quantity is used, and the lesser quantity, with the Numeral Sign, follows close up. Examples:

Exceptions to this rule occur in writing Decimal Coinage (see page 5) and in expressing a definite point of Time when the Colon (dots 2-5) should be placed between the hours and minutes and the Numeral Sign should not be repeated. Example:

ELEMENTARY SIGNS OF OPERATION

(For additional signs used in advanced mathematics, see page 25.)

It is recommended that Signs of Operation not be used in literary Braille books of below the junior high school level.

Plus Sign + \vdots (dots 2-6). Examples:

Minus Sign — : (dots 3-5). Examples:

Plus-or-Minus Sign \pm (dots 2-6, 3-5). Examples:

$$20 \pm 10$$
 : $x \pm y$: [19]

Minus-or-Plus Sign = (dots 3-5, 2-6). Examples:

$$50 \pm 30$$
 : \vdots :

Multiplication Sign × (dots 1-6). Examples:

$$60 \times 5$$
 : 8x × 7y : ... : ...

Division Sign ÷ : (dots 3-4, 3-4). Examples:

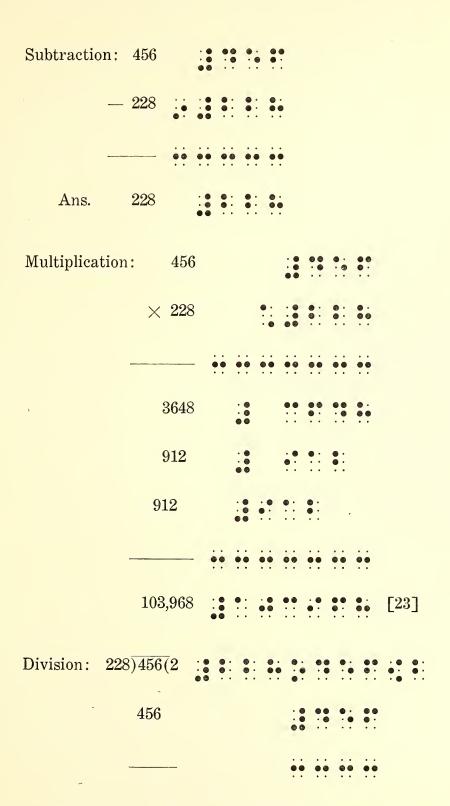
The symbols)...(: (dots 1-3-5 and 2-4-6) may be used when it is desired to place the divisor before the dividend. For example:

It is recommended that this form not be used in algebraic division, as the letter "o" and the "ow" sign are frequently met in such problems.

As an aid in illustrating the meaning of such terms as "numerator," "denominator," "over," etc., fractions and problems in division may be set up to follow the ink-print form. This practice may also be followed in embossing complicated division problems in algebra. It is recommended, however, that the

student be encouraged to master the use of the authorized Division Sign as soon as possible, so that the problems may be worked out on a continuous Braille line on the slate or Braille-writer. Examples:

Note: For use of the continuous line of dots 2-5, 2-5, etc., etc., to indicate the division line in the examples above, see the General Sign of Operation, below. (See also the Double General Sign of Operation, page 14.)



Care should be taken that the General Sign of Operation occupy not less than four Braille cells, lest it be confused with the Equality Sign.

Note: Students wishing to save time in working problems may substitute a skipped line in place of the General Sign of Operation, similar to the practice in working with an arithmetic slate. In such instances, the Numeral Signs and other Signs of Operation may also be omitted if their presence is readily understood.

Double General Sign of Operation (dots 2-3-5-6, 2-3-5-6, 2-3-5-6,

etc.) When it is desired to employ the ink-print set-up in embossing complicated problems in algebra composed of fractions or mixed numbers in either the numerator or [24] denominator, it is suggested that the Double General Sign of Operation (or double line) be used to represent the main division line of the problem. As with the Single Sign of Operation, the sign should occupy not less than four Braille cells. Examples:

This sign may also be used to represent the double line found at the end of balance sheets and financial statements in texts on bookkeeping and commercial arithmetic, as well as the double [25] line at the top of statistical tables.

```
Ratio Sign: (dots 2-5). Examples:
```

```
4:5 : x:y ::::
```

Usages:

In writing lengthy expressions, formulas or equations requiring more than one Braille line, division should be made at a Sign of Operation, preferably following the sign. Example:

```
2,000,000 \times 200 = 200 = 200 \times 200 = 200,000,000 \times 200 \times 20
```

Although the division of short expressions is not recommended, when it is [26] necessary to split an expression composed of the product of several factors, a Multiplication Sign must be inserted before the break. Example:

The expression 15xyz when split after the first letter should be written:

```
yz ......
```

If a Sign of Operation is used in an expression containing words or abbreviations, the symbol should be preceded and followed by a space. However, when only figures are used (including figures written with Symbols or Abbreviations of Value or Measurement) the Sign of Operation is written without a space preceding or following. Examples:

SYMBOLS OF GROUPING

If possible, expressions within Symbols of Grouping should be written complete on one line.

Parentheses ()
$$\vdots$$
 (dots 2-4-6 and 1-3-5). Example: $(x + y)$ \vdots \vdots \vdots \vdots

If more than three symbols of grouping are required, the Literary Parentheses may be used. The Vinculum is not represented in Braille. [28]

Usages:

If an expression composed of the product of several quantities contained within Symbols of Grouping

must be split into two Braille lines, the division should come between the Symbols of Grouping, and a Multiplication Sign should be inserted at the end of the first line. Example:

The expression (a - b) (c - d) when split after the first quantity, should be written:

$$(\mathbf{a} - \mathbf{b}) \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \\ (\mathbf{c} - \mathbf{d}) \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

Examples of Use:

$$[x + (3x - 2y) + z] = 12$$

$$\{2x + [y + (x - y) - z] + 2y\} = 16$$

$$(x - \{3y - [x - 2z] - x\} + 3z) = 5$$

$$\begin{cases} 2x + [y + (x - y) - z] + 2y \\ 3y + [y + (x - y) - z] + 3z \\ 3y + [y + (x - y) - z$$

See also rules for Mixed Numbers, pages 18-19; Index Signs, pages 19-20; Root Signs, pages 23-25; Factorial Sign, page 28; Illustration in Combinations, [29] page 30; and Integral Sign, page 32.

FRACTIONS

Fraction Line / : (dots 3-4). The Fraction Line is placed between the numerator and denominator of a fraction. The denominator is written without a Numeral Sign when both numerator and denominator are simple numerals. For example:

The following way of writing fractions is sometimes used:

Care should be exercised in using this form, as the last figure of the denominator might be mistaken for a Sign of Operation or Punctuation. [30]

MIXED NUMBERS

Usages:

The fractional part of a mixed number is joined to the whole number by a Hyphen. For example:

$$4\frac{7}{8}$$

Whenever a numerator or a denominator consists of more than one term, it must be inclosed in parenthetical marks. For example:

It is usually not necessary to inclose a numerator or a denominator when it is the product of several factors. For example:

It is, however, sometimes an aid to clearness to inclose a term composed of a large number of factors. For example: [31]

When a fraction is employed as the coefficient of a quantity, it must be inclosed. For example:

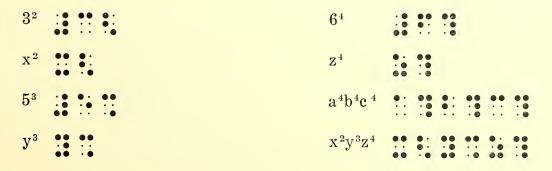
In arithmetic, when a fraction is to be multiplied by a simple numeral, the fraction is not inclosed. For example:

$$\frac{6}{7} \times 5$$

INDEX SIGNS

Numerical Index Sign : (dot 4). The Numerical Index Sign changes all the first ten Braille signs of the alphabet that appear in the index into numerals, unless they are preceded by the Multiplication Sign or by the Letter Sign. For [32] example:

The Index Sign is not used for the Second, Third and Fourth Positive Powers, since special symbols have been assigned to these powers. For example:



Literal Index Sign : (dots 4-5). Examples:

$$7^{m}$$
 : x^{m} : x^{m

Special Usages:

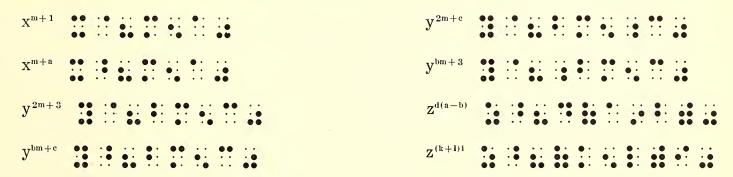
If an exponent relates to a monomial term that is composed of more than one factor, the term is inclosed in Parentheses. For example:

When a polynomial has an exponent, it is inclosed in Parentheses. For example:

A fraction that has an exponent is inclosed in Parentheses. For example:

A monomial exponent is inclosed in Braces, unless it is a simple numeral or a single letter. For example:

A polynomial exponent is inclosed in Braces. For example:



A fractional exponent is inclosed in Braces. For example:

(See also Special Usages under Suffix Signs, page 21.)

SUFFIX SIGNS

Numerical Suffix Sign : (dot 6). The Numerical Suffix Sign changes all the first ten Braille signs of the alphabet that appear in the suffix into numerals, unless they are preceded by the Multiplication Sign or by the Letter Sign. For example:

```
C6 .... X2 .... ...
```

To save space, x1, x2, x3, etc., are sometimes written:

These symbols are then separated from subsequent Punctuation Signs by dot 3, as follows:

```
X_1, \dots, X_3: \dots  X_4: \dots  X_4: \dots
```

Literal Suffix Sign : (dots 5-6). Examples:

Special Usages: When a factor that has one of the first ten signs of the alphabet as an exponent or a suffix is followed in the same term by a factor that is itself one of the first ten signs of the alphabet, the Multiplication Sign is inserted between the two factors. For example:

PRIME SIGNS

ROOT SIGNS

Root Sign (dots 1-2-4-5-6). Examples:



Since the Square Root and the Cube Root are the roots most commonly used, special symbols, which take the place of both Index and Root Sign, have been assigned to them. For example: [38]



Special Usages:

The quantity under the Root Sign is inclosed in Parentheses, unless it is a simple numeral or a single letter, as in the preceding examples. The use of the Parentheses is shown in the following illustrations:

A polynomial, when under a Root Sign, is always inclosed in Parentheses. For example:

Sometimes more than one Symbol of Grouping is required. For example:

$$\sqrt[3]{5x + \sqrt{4x + 3y + z}} = 3$$

$$\sqrt[3]{4x + \sqrt[3]{5x + \sqrt{4x + 3y + z}}} = 2$$

$$\sqrt[3]{9y + \sqrt[3]{5x + \sqrt{x + 6z + y + z}}} = 2$$

$$\sqrt[3]{x + 2y + \sqrt{x^2 - 2xy + y^2}} = \sqrt{2x + y}$$

$$\sqrt[3]{x + 2y + \sqrt{x^2 - 2xy + y^2}} = \sqrt{2x + y}$$

When an entire fraction is under a Root Sign, it is inclosed in Parentheses. For example:

$$\sqrt{\frac{2}{3}}$$

$$\sqrt[3]{7\frac{5}{8}}$$

$$\sqrt[4]{\frac{x}{y}}$$

$$\sqrt[4]{\frac{a+b}{x-y}}$$

$$\sqrt[3]{\frac{a+b}{x-y}}$$

$$\sqrt[3]{\frac{a+b}{x-y}}$$

Compare the meanings of the following fractions:

ADVANCED SIGNS OF OPERATION

(For elementary mathematical Braille signs, see pages 10-16.)

Note: The **Letter Signs** and spaces are frequently omitted, as the possibility of a mistake in interpretation is negligible. Example:

$$p = q$$

Identically Equal To, or Congruent To = (dots 2-3-5-6, 2-3-5-6). Examples:

$$x \times \frac{y}{x} = y$$

The spaces before and after this symbol are frequently omitted when used in algebra. For example: [41]

$$x \equiv y \quad \vdots \quad \vdots \quad \vdots$$

This is frequently written

Not Equal To \neq (dots 2-5, 2-4-5-6). Example: $c \neq d$: \vdots : \vdots

This is frequently written :: ;; ; , but the spaces should not be omitted if there is any possibility of the symbols meaning "the ratio of c to wd."

This is frequently written ..., except when the symbols might be mistaken for "the ratio of p to cq."

This is frequently written :: :: ::

Greater Than or Equal To \geqslant \vdots (dots 2-5, 1-2-4-5). Example:

```
x > y :: :: :: :: :: :: :: ::
```

This is frequently written to be taken lest it be interpreted as "the ratio of x to gy."

This is frequently written :: :: ::

Equal To or Less Than \gtrsim ... (dots 2-5, 2-3-5-6). Example: $p \gtrsim q$

Greater Than or Less Than > < \vdots \vdots (dots 2-5, 1-3-4-6). Example:

This may be written :: ... , but care should be taken lest it be read "the ratio of v to xy."

Greater Than, Equal To, or Less Than > = < (dots 2-5, 1-2-3-4-5-6). Example:

This is often written :: [44]

This is often written ;; ;; , but care is necessary to avoid misinterpretation as "the ratio of x to vy."

INFINITY

FACTORIALS

Factorial Sign (dots 2-3-5-6). The Factorial Sign changes the first ten signs of the alphabet into numerals, unless they are preceded by the Multiplication Sign or by the Letter Sign. For example:

LOGARITHMS

Logarithm Sign log : (dots 4-5-6 and 1-2-3). Example:

Log 50 is sometimes written : , but if the base is indicated, the number must be inclosed between the two parts of the sign. For example:

The Abbreviation "log" written without a period, is sometimes used to mean "the log of." For example:

The Abbreviations "colog" and "antilog" are also used in this manner.

Bar Sign (dots 4-5-6). This [46] sign is used to indicate that the characteristic of a logarithm is negative, but the mantissa is positive. For example:

Since this symbol is only used with the integral part of a decimal number there should be no confusion with the sign for a Recurring Decimal.

PERMUTATIONS AND COMBINATIONS

Permutations. Examples:

Combinations. Examples:

 $^{\rm n}{\rm C_r}$ (number of Combinations of n things taken r together). :

 $^{10}\mathrm{C}_5$: : : : : : : [47]

Illustration in Combinations:

Out of eight consonants and four vowels, how many combinations of letters can be chosen containing two vowels and three consonants? Solution:

TRIGONOMETRICAL RATIOS

 The following Abbreviations may also be substituted for the symbols given above:

```
Sine sin (dots 2-3-4, 3-5).

Cosine cos (dots 1-4, 1-3-5, 2-3-4).

Tangent tan (dots 2-3-4-5, 1, 1-3-4-5).

Cotangent cot (dots 1-4, 1-3-5, 2-3-4-5) or (dots 1-4, 2-3-4-5, 1-3-4-5).

Secant sec (dots 2-3-4, 1-5, 1-4).

Cosecant cosec (dots 1-4, 2-3-4, 1-4).
```

The Index of the power to which a Trigonometrical Ratio is raised is inserted between the sign of the ratio and the angle. For example:

```
\frac{\sin^3 x}{\cos^4 x} \stackrel{\text{\tiny i.i.}}{\text{\tiny i.i.}} \stackrel{\text{\tiny i.i.}}
```

HYPERBOLIC FUNCTIONS

```
      Hyperbolic Sine sinh
      (dots 2-3-5, 2-3-4, 1-2-5).

      Hyperbolic Cosine cosh
      (dots 2-3-5, 1-4, 1-2-5).

      Hyperbolic Tangent tanh
      (dots 2-3-5, 2-3-4-5, 1-2-5).

      Hyperbolic Contangent coth
      (dots 2-3-5, 1-2-5-6, 1-2-5).

      Hyperbolic Secant sech
      (dots 2-3-5, 3-6, 1-2-5).

      Hyperbolic Cosecant cosech
      (dots 2-3-5, 1-2-6, 1-2-5).
```

The following Abbreviations may also be substituted for the symbols given above:

Hyperbolic Sine sinh :: :: (dots 2-3-4, 3-5, 1-2-5).

Hyperbolic Cosine cosh ... (dots 1-4, 1-3-5, 2-3-4, 1-2-5).

Hyperbolic Tangent tanh : (dots 2-3-4-5, 1, 1-3-4-5, 1-2-5).

Hyperbolic Cotangent coth (dots 1-4, 1-3-5, 2-3-4-5, 1-2-5) or ctnh (dots 1-4, 2-3-4-5, 1-3-4-5, 1-2-5).

Hyperbolic Secant sech : (dots 2-3-4, 1-5, 1-4, 1-2-5).

Hyperbolic Cosecant cosech :: :: :: :: (dots 1-4, 1-3-5, 2-3-4, [51] 1-5, 1-4, 1-2-5) or csch :: :: :: (dots 1-4, 2-3-4, 1-4, 1-2-5).

Note: Care should be taken not to use the ch or sh or th signs in the above Abbreviations.

SYMBOLS FOR CALCULUS

Differential Coefficient: $\frac{dx}{dt}$: \vdots : \vdots

Second Differential Coefficient: $\frac{d^2x}{dt^2}$: $\frac{d^2x}{dt^2}$: $\frac{d^2x}{dt^2}$: $\frac{d^2x}{dt^2}$

Integral Sign \int \vdots \vdots (dots 3, 2-3-4). Examples:

[52] ABBREVIATIONS

Abbreviations employed in ink-print mathematical texts may be used in Braille. In addition, the list below may be used in texts more advanced than arithmetic. Some texts may employ abbreviations which vary slightly from the form given here, and the Abbreviation Point may or may not be used. Abbreviations should be preceded and followed by a space:

adjacent		circle 🔾	• • • · · · · · · · · · · · · · · · · ·
alternate	•: •: ••	circular measure	
altitude	· · · · · · · · · · · · · · · · · · ·	circum	
angle ∠	: • •: (dots 4-5-6, 1)	circumference \odot	
angular	(dots 4-5-6, 1, 1-2-3-5)	complement ::	• • • • • • • • • • • • • • • • • • •
another	• • • • • • • • • • • • • • • • • • •	complementary _ ::	• • • •
arc*	· • • • • • • • • • • • • • • • • • • •	condition •••	• • • • • • • • • • • • • • • • • • • •
axiom	• • • • • • • • • • • • • • • • • • • •	consider	· • • • • • • • • • • • • • • • • • • •
base	: • •: (dots 4-5, 1-2)	constant	· • · ·
bisect	: (dots 4-5-6, 1-2) [53]	construct	
call	:• •• (dots 4-5, 1-4)	construction	
center	: dots 4-5-6, 2-5)	corollary	• • • • • • • • • • • • • • • • • • • •
centric	dots 4-5-6, 2-5, 1-4)	correspond •••	
change	: • : (dots 4, 1-6)	curve	0 · 0 · 0 · 0 · 0 · 0 · 0 · 0 · 0 · 0 ·
chord	: • • : (dots 4-5-6, 1-6)	definition	• • • • • • • • • • • • • • • • • • • •

^{*}Not an abbreviation.

depend	• • • • • • • • • • • • • • • • • • • •	general	• · · · • · · · · · · · · · · · · · · ·
describe	• • • • • • • • • • • • • • • • • • • •	geometrical	00 00 · 0 0 · 0 · 0 · 0 · 0 · 0 · 0 · 0
diagonal	[54]	geometry	00 00 ·0 00 00 ·· 00 ·0 ·· 0 · 0 ·0
diameter	:• •• (dots 4-5, 1-4-5)	graph	: • • • (dots 4-5-6, 1-2-4-5)
differ	• • • • • • • • • • • • • • • • • • • •	horizontal	0 · 0 · 0 · · · · · · · · · · · · · · ·
different	• • • • • • • • • • • • • • • • • • •	hypotenuse	0 · 0 0 0 · 0 · 0 · 0 · 0 · 0 · 0 · 0 ·
direction	(dots 4-5-6, 1-4-5)	hypothesis	0 · 0 0 0 · 0 · 0 · 0 · 0 · 0 · 0 · 0 ·
divide	• • • • • • • • • • • • • • • • • • •	identity	[55]
equal	• · • • • · · · · · · · · · · · · · · ·	infinite	· · • • · · · · · · · · · · · · · · · ·
equation	• • • • • • • • • • • • • • • • • • •	inscribe	· · · • • • • · · · · · · · · · · · · ·
equidistant	• · • • • · • · • · · • · · · • · · · ·	intercept	- · · · · · · · · · · · · · · · · · · ·
equilateral	• · • • • • · • · · · · · · · · · · · ·	interior	· · · · • · · · · · · · · · · · · · · ·
example	(dots 4-5-6, 1-5)	internal	· · · • • · · · · · · · · · · · · · · ·
exercise	• · • • · · · · · • •	intersect	· · · · · · · · · · · · · · · · · · ·
expression	· · · · · · · · · · · · · · · · · · ·	intersection	· · · • • • • • • • • • • • • • • • • •
exterior	0 · 00 · 0 · 0 · · · · · · · · · · · ·	isosceles	
external	• · • • • • · · · · · · · · · · · · · ·	length	0 · 00 0 · · 0 0 · · 0
figure	• • • • • • • • • • • • • • • • • • •	line	: • • : (dots 4-5-6, 1-2-3)
follow	: • • • (dots 4-5-6, 1-2-4)	logarithm	(See also page 29).

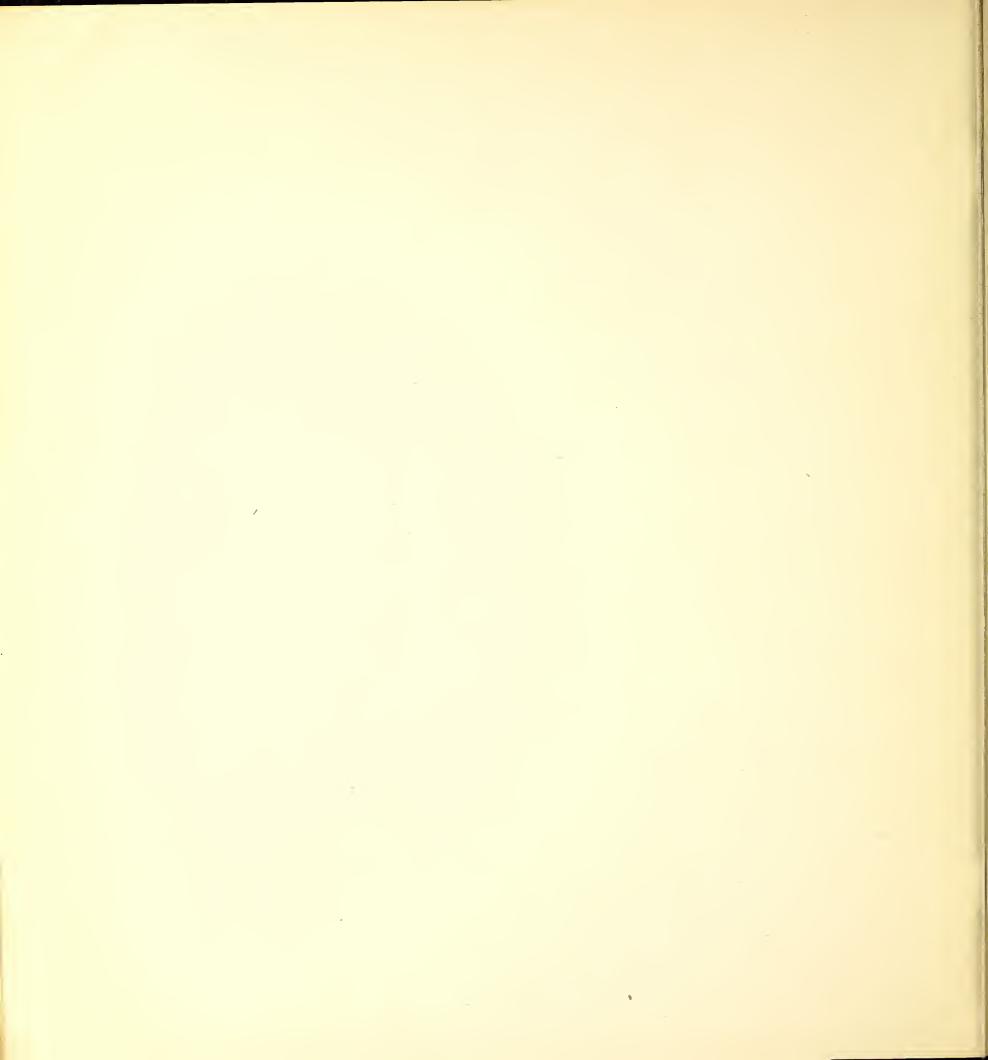
magnitude	•• • • • • • • • • • • • • • • • • • • •	projection	•• •· •• •• •· •• • • •
maximum	•• •• •• ••• •·	property	• • • • • • • • • • • • • • • • • • •
measure	•• •• •• •• •• ••	proportion	(See also page 15).
meter	•••••	proposition	•• •• •• •• •• •• •• ••
middle	: • • • (dots 4-5, 1-3-4)	quadrilateral	•• •• • • • • • • • • • • • • • • • •
negative		quantity	•• •• •• ••
number	(dots 4-5-6, 1-3-4-5)	quantities	•• •• •• ••
opposite	· · · · · · · · · · · · · · · · · · ·	radians	• • • • • • • • • • • • • • • • • • •
origin	(dots 4-5-6, 1-3-5)	radius	• • • • • • • • • • • • • • • • • • •
parallel	•• •• ••	ratio	(dots 4-5-6, 1-2-3-5) (See also page 15).
parallelogram	•• •• •• ••	rectangle 🗆	• • • • • • • • • • • • • • • • • • •
particular	(dots 5, 1-2-3-4, 1-2-3-5) [56]	rectilineal	• • • • • • • • • • • • • • • • • • •
perpendicular \perp		rectilinear	• • • • • • • • • • • • • • • • • • •
plane		represent	• • • • • • • • • • • • • • • • • • •
point	(dots 4-5-6, 1-2-3-4)	require	• • • • • • • • • • • • • • • • • • •
portion	• • • • • • • • • • • • • • • • • • • •	required	[57]
position	•• •• •• •• •• ••	respective	• · · • • • · • · • · · · · · · · · · ·
positive	• • • • • • • • • • • • • • • • • • •	respectively	• • • • • • • • • • • • • • • • • • •
postulate	•• •• ••	right	(dots 5, 1-2-3-5)
problem	•• ••	right angle L	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

satisfy	· • • • • • • • • • • • • • • • • • • •	theorem	•• ••
side	(dots 4-5, 2-3-4)	therefore	• • • • • • • • • • • • • • • • • • • •
similar	· • • • • • • • • • • • • • • • • • • •	together	
square 🗌	• • • • • • • • • • • • • • • • • • • •	triangle \triangle	· · · · · · · · · · · · · · · · · · ·
straight*	· • · • • • • • • • • • • • • • • • • •	trigonometrical	[58]
supplement	· • • • • • • • • • • • • • • • • • • •	value	(dots 4-5-6, 1-2-3-6)
supplementary		vertex	• · • • • • • • • • • • • • • • • • • •
surface	· • • · • · • · · · · · · · · · · · · ·	vertical	· · · · · · · · · · · · · · · · · · ·
symmetrical	· • • • • • · • · • · • · • · • · • · •	volume	• · • · • · • · • · • · • · • · • · • ·
symmetry		wherefore	•••••
tangent :	(geometrical) (See also form		
page 3	30, and form :: :: ,		
page 31).			

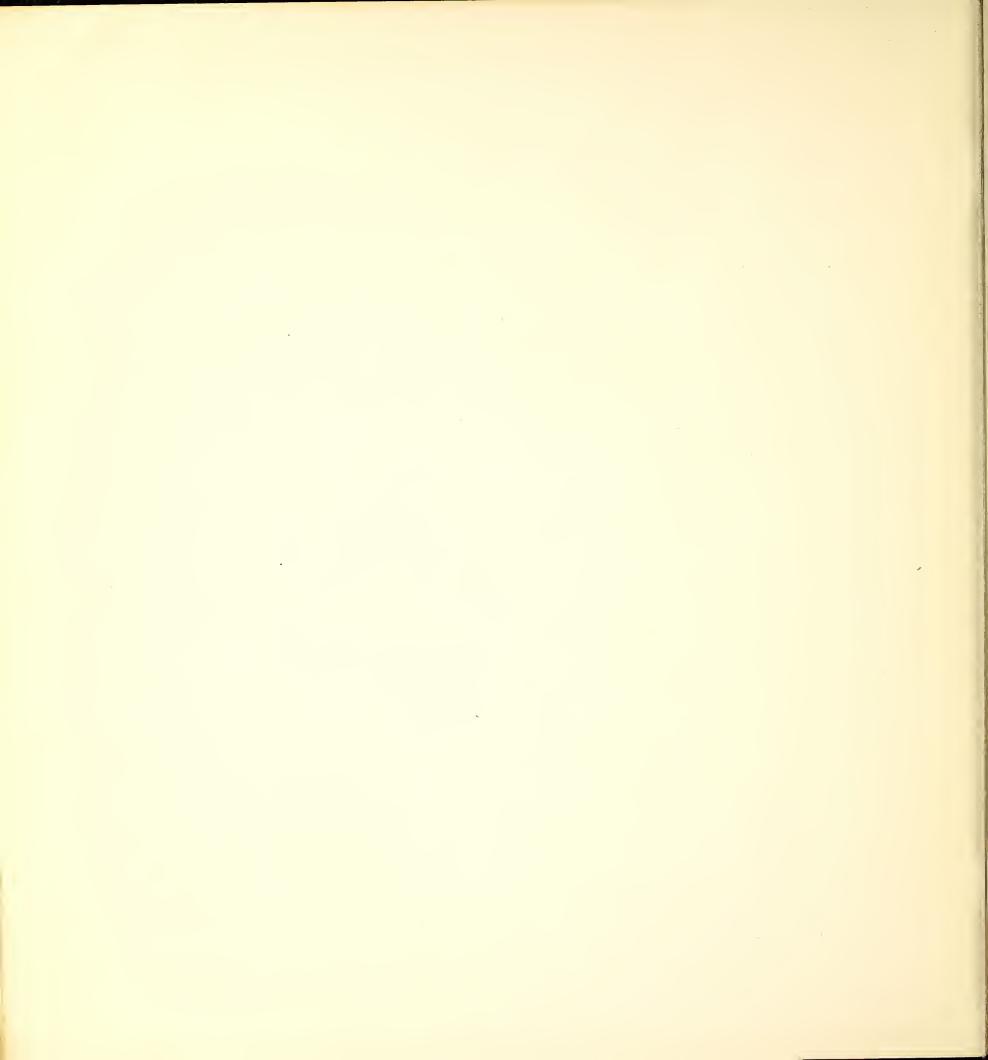
^{*}Also, when grade 2 is not used.

GREEK LETTERS

Alpha	A	а	• • • •	Nu	N	ν	•	••
Beta	В	β		Xi	囯	ξ	• •	•••
Gamma	Г	γ		Omicron	0	o	• •	• •
Delta	Δ	δ		Pi	п	π	•	• •
Epsilon	E	€		Rho	P	ρ	• •	•••
Zeta	Z	ζ		Sigma	Σ σ	or s	• •	•:
Eta	Н	η		Tau	Т	au	• •	•••
Theta	Θ	θ		Upsilon	Υ	υ	• :	• :
Iota	I	ι	:: :• •: •:	Phi	Φ	φ	• •	• •
Kappa	K	κ	· · · · · · · · · · · · · · · · · · ·	Chi	X	χ	· :	
Lambda	Λ	λ	· · • · · · · · · · · · · · · · · · · ·	Psi	Ψ	ψ	• •	• • •
Mu	M	μ		Omega	Ω	ω	• •	











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TAYIOR, H.M.
Braille mathematical notation
2nd American revision, 1942.

Date Due						
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